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SOME PROPERTIES OF CIRCLES AND RELATED CONICS.

BY JAMES H. WEAVER.

Let there be given two circles C_1 and C_2 with centers O_1 and O_2 respectively. The locus of the center of a circle tangent to C_1 and C_2 will be two confocal conics, E_1 and H_1 , with foci O_1 and O_2 . Conversely, with every two confocal conics there may be associated two circles having the common foci as centers, and with radii r_1 and r_2 , such that $r_1 + r_2 = 2a$ and $r_2 - r_1 = 2a'$, where a and a' denote the semi-major axes of the two confocals.

Theorem I: If from any point P on the radical axis of the two circles C_1 and C_2 , tangents be drawn to C_1 , C_2 , E_1 and H_1 , the points of contact lie on four straight lines, two of which pass through O_1 and two through O_2 .

Proof: Let the points of contact of the tangents to C_1 and C_2 be B , B' , A , A' and to E_1 be D and D' (See Figure).

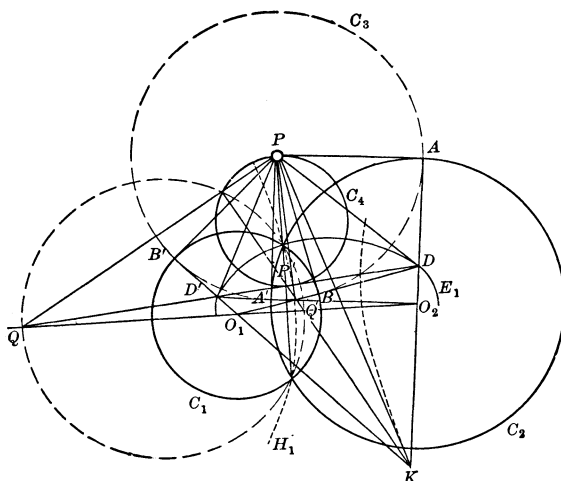


FIG. 1.

Since P is on the radical axis of C_1 and C_2 , $PA = PB = PA' = PB'$, and the four points A , B , A' and B' lie on a circle C_3 with center P , and which cuts C_1 and C_2 orthogonally. Therefore O_1B and O_2A are tangent to C_3 . Let O_1B and O_2A intersect in D_1' . Then D_1' is on the conic E_1 because it is the center of a circle tangent to C_1 and C_2 . But PD_1' bisects

the angle formed by O_1B and O_2A . It is therefore tangent to E_1 , and $D'_1 = D$. Therefore D and A are collinear with O_2 and D and B with O_1 . Similarly it may be shown that all the points of contact of the tangents from P lie on the lines O_2A , O_2A' , O_1B and O_1B' .

The two following theorems follow immediately.

Theorem II: The four points of contact of the tangents from P to E_1 and H_1 determine a quadrangle the vertices of whose diagonal triangle are the centers of C_1 and C_2 and P' the conjugate of P with respect to the system.*

Theorem III: The four lines on which the points of contact lie determine a quadrilateral, the sides of whose diagonal triangle are the line of centers of C_1 and C_2 and the polars of P with respect to E_1 and H_1 .

Since this triangle is self polar with respect to C_3 , P is its ortho center. The two following theorems may be easily proved analytically.

Theorem IV: The polars of P with respect to E_1 and H_1 pass through the centers of perspective of C_1 and C_2 .

Theorem V: The perpendiculars from P to the polars of E_1 and H_1 intersect these polars on the circle of similitude of C_1 and C_2 .

Moreover a circle C_4 on PP' as diameter will pass through this point of intersection. We therefore have

Theorem VI: The polars of P with respect to E_1 and H_1 intersect the polars of P' with respect to H_1 and E_1 respectively on the circle with PP' as diameter.

* The polars of P with respect to C_1 , C_2 , E_1 and H_1 intersect in P' on the radical axis of C_1 and C_2 . This may be easily proven analytically.